Inverse Trigonometric Functions

Inverse Function

If y = f(x) and x = g(y) are two functions such that f (g(y)) = y and g (f(y)) = x, then f and y aresaid to be inverse of each other i.e. $g = f^{-1}$

IF y = f(x), then $x = f^{-1}(y)$

Inverse Trigonometric Functions

If y = sin X, then $x = \sin^{-1}y$, similarly for other trigonometric functions.

This is called inverse trigonometric function .

Now, $y = \sin^{-1}(x)$, $y \in [\pi / 2 , \pi / 2]$ and $x \in [-1,1]$.

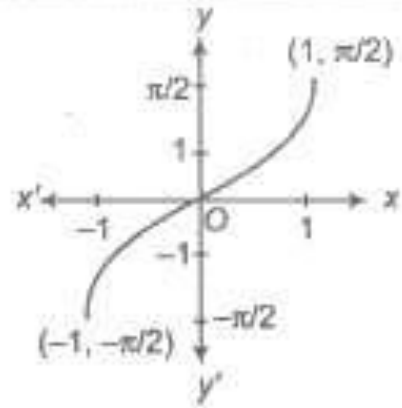
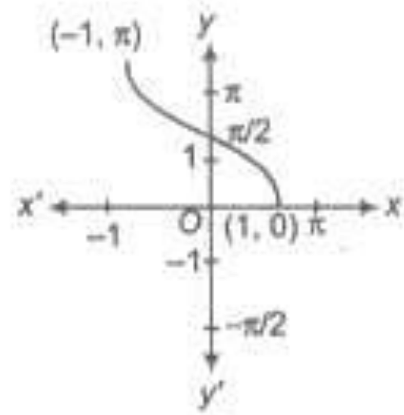
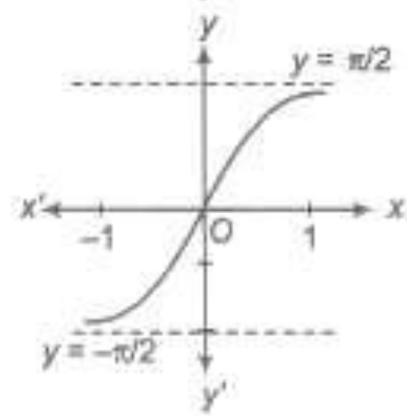
(i) Thus, $\sin^{-1}x$ has infinitely many values for given $x \in [-1, 1]$.

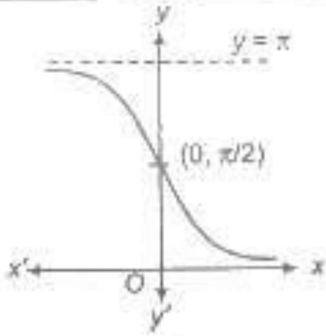
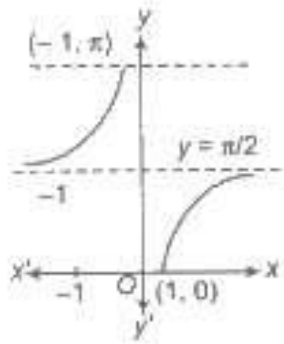
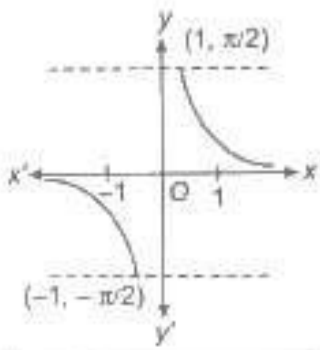
(ii) There is only one value among these values which lies in the interval $[\pi / 2 , \pi / 2]$. Thisvalue is called the principal value.

Domain and Range of Inverse Trigonometric FunctionsGraphs of Inverse

Trigonometric Functions

Functions	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$
$\sec^{-1} x$	$R-(-1, 1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$\operatorname{cosec}^{-1} x$	$R-(-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$

S. No.	Function	Graph
1.	$y = \sin^{-1} x$	
2.	$y = \cos^{-1} x$	
3.	$y = \tan^{-1} x$	

<table>
<tr><th>S. No.</th><th>Function</th><th>Graph</th></tr>
<tr><td>4.</td><td>$y = \cot^{-1} x$</td><td>
</td></tr>
<tr><td>5.</td><td>$y = \sec^{-1} x$</td><td>
</td></tr>
<tr><td>6.</td><td>$y = \operatorname{cosec}^{-1} x$</td><td>
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Properties of Inverse Trigonometric Functions

Property I

(i) $\sin^{-1}(\sin\theta) = \theta;$ $\quad$ if $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1}(\cos\theta) = \theta;$ $\quad$ if $\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta;$ $\quad$ if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta;$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \theta \neq 0$

(v) $\sec^{-1}(\sec\theta) = \theta;$ $\quad$ if $\theta \in [0, \pi], \theta \neq \frac{\pi}{2}$

(vi) $\cot^{-1}(\cot\theta) = \theta;$ $\quad$ if $\theta \in (0, \pi)$

Property II

(i) $\sin(\sin^{-1} x) = x$;   if $x \in [-1, 1]$

(ii) $\cos(\cos^{-1} x) = x$;   if $x \in [-1, 1]$

(iii) $\tan(\tan^{-1} x) = x$;   if $x \in R$

(iv) $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec(\sec^{-1} x) = x$;   if $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot(\cot^{-1} x) = x$;   if $x \in R$

Property III

(i) $\sin^{-1}(-x) = -\sin^{-1} x$;   if $x \in [-1, 1]$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$;   if $x \in [-1, 1]$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x$;   if $x \in R$

(iv) $\operatorname{cosec}^{-1}(-x) = \pi - \operatorname{cosec}^{-1} x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x$;   if $x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x$;   if $x \in R$

Property IV

(i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$; if $x \in (-\infty, -1] \cup [1, \infty)$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x; & \text{if } x > 0 \\ -\pi + \cot^{-1} x; & \text{if } x < 0 \end{cases}$

Property V

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$; if $x \in [-1, 1]$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$; if $x \in R$

(iii) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$; if $x \in (-\infty, -1] \cup [1, \infty)$

Property VI

(i) $\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \\ \pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}; \end{cases}$

if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$ or if $xy < 0$ and $x^2 + y^2 > 1$
if $0 < x, y \le 1$ and $x^2 + y^2 > 1$
if $-1 \le x, y < 0$ and $x^2 + y^2 > 1$

(ii) $\sin^{-1} x - \sin^{-1} y = \begin{cases} \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \\ \pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \\ -\pi - \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}; \end{cases}$

if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$ or if $xy > 0$ and $x^2 + y^2 > 1$
if $0 < x \le 1, -1 \le y \le 0$ and $x^2 + y^2 > 1$
if $-1 \le x < 0, 0 < y \le 1$ and $x^2 + y^2 > 1$

Property VII

(i) $\cos^{-1} x + \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le x, y \le 1 \text{ and } x + y \ge 0 \\ 2\pi - \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}; & \text{if } -1 \le x, y \le 1 \text{ and } x + y \le 0 \end{cases}$$

(ii) $\cos^{-1} x - \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}; \text{if } -1 \le x, y \le 1 \text{ and } x \le y \\ -\cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}; \text{if } -1 \le y \le 0, 0 < x \le 1 \text{ and } x \ge y \end{cases}$$

Property VIII

(i) $\tan^{-1} x + \tan^{-1} y$

$$= \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right); & \text{if } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right); & \text{if } x > 0, y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right); & \text{if } x < 0, y < 0 \text{ and } xy > 1 \end{cases}$$

(ii) $\tan^{-1} x - \tan^{-1} y$

$$= \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right); & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right); & \text{if } x > 0, y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right); & \text{if } x < 0, y > 0 \text{ and } xy < -1 \end{cases}$$

Property IX

(i) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}} = \cot^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$$

(ii) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$

$$= \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\left(\frac{1}{x}\right)$$

$$= \operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

(iii) $\tan^{-1} x = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right) = \cot^{-1}\left(\dfrac{1}{x}\right)$

$$= \operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right) = \sec^{-1}(\sqrt{1+x^2})$$

Property X

(i) $2\sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -\dfrac{1}{\sqrt{2}} \le x \le \dfrac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } \dfrac{1}{\sqrt{2}} \le x \le 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}); & \text{if } -1 \le x \le -\dfrac{1}{\sqrt{2}} \end{cases}$

(ii) $2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2-1); & \text{if } 0 \le x \le 1 \\ 2\pi - \cos^{-1}(2x^2-1); & \text{if } -1 \le x \le 0 \end{cases}$

(iii) $2\tan^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } -1 < x \le 1 \\ \pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } x > 1 \\ -\pi + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); & \text{if } x < -1 \end{cases}$

Property XI

$$
\text{(i)}\quad 3\sin^{-1}x=\begin{cases}\sin^{-1}(3x-4x^3); & \text{if } -\frac{1}{2}\le x\le \frac{1}{2}\\ \pi-\sin^{-1}(3x-4x^3); & \text{if } \frac{1}{2}< x\le 1\\ -\pi-\sin^{-1}(3x-4x^3); & \text{if } -1\le x< -\frac{1}{2}\end{cases}
$$

$$
\text{(ii)}\quad 3\cos^{-1}x=\begin{cases}\cos^{-1}(4x^3-3x); & \text{if } \frac{1}{2}\le x\le 1\\ 2\pi-\cos^{-1}(4x^3-3x); & \text{if } -\frac{1}{2}\le x\le \frac{1}{2}\\ 2\pi+\cos^{-1}(4x^2-3x); & \text{if } -1\le x\le -\frac{1}{2}\end{cases}
$$

$$
\text{(iii)}\quad 3\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } -\frac{1}{\sqrt{3}}< x< \frac{1}{\sqrt{3}}\\ \pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } x>\frac{1}{\sqrt{3}}\\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right); & \text{if } x< -\frac{1}{\sqrt{3}}\end{cases}
$$

Property XII

$$
\text{(i)}\quad 2\tan^{-1}x=\begin{cases}\sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } -1\le x\le 1\\ \pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } x>1\\ -\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right); & \text{if } x<-1\end{cases}
$$

$$
\text{(ii)}\quad 2\tan^{-1}x=\begin{cases}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); & \text{if } 0\le x<\infty\\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); & \text{if } -\infty< x\le 0\end{cases}
$$

Important Results

(i) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x + y + z - xyz}{1 - xy - yz - zx}\right)$

(ii) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then $xy + yz + zx = 1$

(iii) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then $x + y + z = xyz$

(iv) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{\pi}{2}$, then $x^2 + y^2 + z^2 + 2xyz = 1$

(v) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then
$x\sqrt{1 - x^2} + y\sqrt{1 - y^2} + z\sqrt{1 - z^2} = 2xyz$

(vi) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx = 3$

(vii) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then $x^2 + y^2 + z^2 + 2xyz = 1$

(viii) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then $xy + yz + zx = 3$

(ix) If $\sin^{-1} x + \sin^{-1} y = \theta$, then $\cos^{-1} x + \cos^{-1} y = \pi - \theta$

(x) If $\cos^{-1} x + \cos^{-1} y = \theta$, then $\sin^{-1} x + \sin^{-1} y = \pi - \theta$

(xi) If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{2}$, then $xy = 1$

(xii) If $\cot^{-1} x + \cot^{-1} y = \frac{\pi}{2}$, then $xy = 1$

(xiii) If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$,

then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$

(xiv) $\tan^{-1} x_1 + \tan^{-1} x_2 + \ldots + \tan^{-1} x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - \ldots}{1 - S_2 + S_4 - S_6 + \ldots}\right)$,

where$S_k$denotes the sum of the product of x_1, x_2,….$x_n$takes k at a time.

Inverse

Trigonometric Equation

An equation involving one or more trigonometrical ratios of unknown angle is called a trigonometric equation .

Solution/Roots of a Trigonometric Equation

A value of the unknown angle which satisfies the given equation, is called a solution or root ofthe equation.

The trigonometric equation may have infinite number of solutions.

(i) **Principal Solution** – The least value of unknown angle which satisfies the given equation,is called a principal solution of trigonometric equation.

(ii) **General Solution** – We know that, trigonometric function are periodic and solution oftrigonometric equations can be generalised with the help of the periodicity of the trigonometricfunctions. The solution consisting of all possible solutions of a trigonometric equation is calledits general solution.

Important Results

(i) $\sin\theta = 0 \Rightarrow \theta = n\pi$

(ii) $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$

(iii) $\tan\theta = 0 \Rightarrow \theta = n\pi$

(iv) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(v) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$

(vi) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vii) $\sin^2\theta = \sin^2\alpha, \cos^2\theta = \cos^2\alpha, \tan^2\theta = \tan^2\alpha$

$\Rightarrow \quad \theta = n\pi \pm \alpha$

(viii) $\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}$

(ix) $\cos\theta = 1 \Rightarrow \theta = 2n\pi$

(x) $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$

(xi) $\left.\begin{array}{l}\sin\theta = \sin\alpha \text{ and } \cos\theta = \cos\alpha \\ \sin\theta = \sin\alpha \text{ and } \tan\theta = \tan\alpha \\ \tan\theta = \tan\alpha \text{ and } \cos\theta = \cos\alpha\end{array}\right\} \Rightarrow \theta = 2n\pi + \alpha$

Important Points to be Remembered

(i) While solving an equation, we have to square it, sometimes the resulting roots does notsatisfy the original equation.

(ii) Do not cancel common factors involving the unknown angle on LHS and RHS.Because itmay be the solution of given equation.

(iii) (a) Equation involving sec θ or tan θ can never be a solution of the form (2n + 1) π / 2.

(b) Equation involving coseca or cote can never be a solution of the form θ = nπ.

MCQ

1. $sin\left(\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

2. The value of $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$ is equal to

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

3. The value of expression $2\ sec^{-1}2 + sin^{-1}\frac{1}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) 1

4. $sin\left(\frac{1}{2}cos^{-1}\frac{4}{5}\right) =$

(a) $\frac{1}{\sqrt{10}}$ (b) -10 (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

5. The domain of the function $\cos^{-1}(2x-1)$ is

(a) [0,1] (b) [-1,1] (c) (-1,1) (d) $[0, \pi]$

6. if $cos^{-1}\alpha + cos^{-1}\beta + cos^{-1} = 3\pi$ then $\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$ equals

(a) 0 (b) 1 (c) 6 (d) 12

7. Domain of the definition of the function $f(x) = \sqrt{sin^{-1}(2x) + \pi/6}$ is

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$ (b) $\left[-\frac{1}{2},\frac{1}{9}\right]$ (c) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (d) $\left[-\frac{1}{4},\frac{1}{4}\right]$

8. Range of f(x)= $\sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is

(a) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ (c) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$ (d) None of these

9. if $sin^{-1}x = \theta + \beta$ and $sin^{-1}y = \theta - \beta$ then 1+xy

(a) $sin^2\theta + sin^2\beta$ (b) $sin^2\theta + cos^2\beta$ (c) $cos^2\theta + cos^2\beta$ (d) $cos^2\theta + sin^2\beta$

10. The value of $sin^{-1}cos\frac{33\pi}{5}$ is

(a) $\frac{3\pi}{5}$ (b) $-\frac{7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{10}$

11. $cos^{-1}\left(cos\frac{7\pi}{6}\right)$ is equal to

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

12. $sin^{-1}\left(sin\frac{2\pi}{3}\right)$ is

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{\pi}{3}$

13. *The value of* $cos^{-1}\left(-sin\frac{7\pi}{6}\right)$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{7\pi}{6}$ (c)$\frac{\pi}{3}$ (d) None of these

14. The value of $cos^{-1}\left(cos\frac{3\pi}{2}\right)$ is equal to

(a)$\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c)$\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

15. if$\pi \leq x \leq 2x$ then $cos^{-1}(\cos x)$ is equal to

(a) x (b) –x (c) $2\pi + x$ (d) $2\pi - x$

16. sin(tan^{-1} x), |x|< 1 equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c)$\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

17. if x takes non positive permissible value , then sin^{-1} x=

(a) $cos^{-1}\sqrt{1-x^2}$ (b) $-cos^{-1}\sqrt{1-x^2}$ (c)$cos^{-1}\sqrt{x^2-1}$ (d) $\pi - cos^{-1}\sqrt{1-x^2}$

18. if$sin^{-1}x = \frac{\pi}{5}$ for some x∈ $(-1,1)$ then the value of $cos^{-1}x$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{5\pi}{10}$ (c)$\frac{7\pi}{10}$ (d) $\frac{9\pi}{10}$

19. $cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(cos\frac{9\pi}{10} - sin\frac{9\pi}{10}\right)\right\} =$

(a) $\frac{3\pi}{20}$ (b) $\frac{7\pi}{20}$ (c)$\frac{7\pi}{10}$ (d) $\frac{17\pi}{20}$

20. The values of $x\ satisfying$ $\tan(sec^{-1}x) = \sin(cos^{-1}\frac{1}{\sqrt{5}})$ is/are

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{3}{\sqrt{5}}$ (c)$-\frac{\sqrt{5}}{3}$ (d) $-\frac{3}{\sqrt{5}}$

21.sec^2(tan^{-1} 2)+cosec^2(cot^{-1} 3)=

(a) 5 (b) 13 (c) 15 (d) 6

22.$sin^{-1}\frac{\sqrt{x}}{\sqrt{x+a}}$ is equal to

(a) $cos^{-1}\sqrt{\frac{x}{a}}$ (b) $cosec^{-1}\sqrt{\frac{x}{a}}$ (c)$tan^{-1}\sqrt{\frac{x}{a}}$ (d) None of these

23. if$x \in \left(\frac{3\pi}{2}, 2\pi\right)$ then value of expression $sin^{-1}(\cos(cos^{-1}(\cos x) + sin^{-1}(\sin x)))$ equals

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$ (c) 0 (d) None of these

24. $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x is equal to

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$

25.$if\ f(x) = sin^{-1}x + cos^{-1}x + tan^{-1}x + cot^{-1}x + sec^{-1}x$then f(x) lies in the interval

(a) $[\pi, 2\pi]$ (b) $(\pi, 2\pi)$ (c) $\left[\pi, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ (d) None of these

26. if$cos^{-1}x + cos^{-1}y = 2\pi$ then $sin^{-1}x + sin^{-1}y$ is equal to

(a) π (b) $-\pi$ (c) $\frac{\pi}{2}$ (d) None of these

27. if 3tan^{-1}x+ cot^{-1}x= π then x equals

(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

28. if$cos\left(sin^{-1}\frac{2}{5} + cos^{-1}x\right) = 0$ then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 0 (d) 1

29. The value of x for which cos^{-1}(cos 4)> 3x^2-4x, is

(a) $\left(0, \frac{2+\sqrt{6\pi-8}}{3}\right)$ (b) $\left(\frac{2-\sqrt{6\pi-8}}{3}, 0\right)$ (c) (-2,2) (d) $\left(\frac{2-\sqrt{6\pi-8}}{3}, \frac{2+\sqrt{6\pi-8}}{3}\right)$

ANSWERS

1	**d**	2	**b**	3	**b**	4	**a**	5	**a**	6	**c**	7	**a**	8	**c**	9	**b**
10	**d**	11	**d**	12	**d**	13	**c**	14	**a**	15	**d**	16	**d**	17	**b**	18	**a**
19	**d**	20	**b**	21	**c**	22	**c**	23	**b**	24	**c**	25	**c**	26	**b**	27	**b**
28	**b**	29	**d**														