Probability

Basic Definitions

Random Experiment:

- An experiment, whose all possible outcomes are known in advance but the outcome of any specific performance cannot predicted before the completion of the experiment
- Eg: Tossing of acoin.

Sample-space

- A set of all possible outcomes associated with same randomexperiment
- Denoted by'S'.
- Eg: In the experiment of tossing a die, If we are interested in the number that shows on the top face, then sample space would be S = {1, 2, 3, 4, 5,6}

Experiment or Trial

- It is a series of action where the outcomes are alwaysuncertain.
- Eg: - Tossing of a coin, Selecting a card from deck of cards, throwing adice.

Event

- Subset of sample –space.
- In any sample space we may be interested in the occurrence of certain events rather than in the occurrence of a specific element in the samplespace.

Simple Event

- If an event is a set containing only one element of thesample-space

Compound Event

- A compound event is one that can be represented as a union of samplepoints
- Eg:
  Event of drawing a heart from a deck of cards is the subset A = {heart} of the sample space S =
  {heart, spade, club, diamond}.
  ThereforeAisasimpleevent.NonetheeventBofdrawingaredcardisacompoundevents ince B = {heart U diamond} = {heart,diamond}.

Probability

- If a random experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes favours toA,
  Then the probability of event A, P (A) = n/N i.e. favourable cases/total no. of cases.
- Remarks:
  - If the probability of certain event is one, it doesn't mean that event is going to happen withcertainty
  - It's just predicting that, the event is most likely to occur in comparison to

otherevents. Predictionsdependuponthepastinformationandofcoursealsoonthewayofanalysing the information athand

- Similarly if the probability of certain event is zero, it doesn't mean that, the event can never occur!

Mutually exclusive Event

- If two events are mutually exclusive they cannot occursimultaneously.

Independent Events

- Events are said to be independent if the occurrence or non-occurrence of one does not affect the occurrence or non-occurrence ofother.

Exhaustive Event

- A set of events is said to be exhaustive if the performance of random experiment always result in the occurrence of at least one ofthem

Conditional Probability

- The conditional probability of an event B is the probability that the event will occur given the knowledge that an event A has alreadyoccurred.
- This probability is written P (B|A), notation for the probability of B givenA.
- In the case where events A and B are independent (where event A has no effect on the probability of event B), the conditional probability of event B given event A is simply the probability of event B, that isP(B).
- If events A and B are not independent, then the probability of the intersection of A and B (the probability that both events occur) is defined by P (A and B) = P (A) P(B|A).

- If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F hasoccurred,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \text{ provided } P(F) \neq 0$$

I.e. P (E|F) is given by

- Eg: If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$$

Properties of conditional probability

Let E and F be events of a sample space S of an experiment, then we have

Property 1

$P(S|F) = P(F|F) = 1$

We know that

$$P(S|F) = \frac{P(S \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Also $$P(F|F) = \frac{P(F \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Thus $P(S|F) = P(F|F) = 1$

Property 2

If A and B are any two events of a sample space S and F is an event of S such that P (F) ≠ 0, then

$$P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap B)|F)$$

In particular, if A and B are disjoint events,

$$P((A \cup B)|F) = P(A|F) + P(B|F)$$

We have

$$P((A \cup B)|F) = \frac{P[(A \cup B) \cap F]}{P(F)}$$

$$= \frac{P[(A \cap F) \cup (B \cap F)]}{P(F)}$$

(by distributive law of union of sets over intersection)

$$= \frac{P(A \cap F) + P(B \cap F) - P(A \cap B \cap F)}{P(F)}$$

$$= \frac{P(A \cap F)}{P(F)} + \frac{P(B \cap F)}{P(F)} - \frac{P[(A \cap B) \cap F]}{P(F)}$$

$$= P(A|F) + P(B|F) - P((A \cap B)|F)$$

When A and B are disjoint events, then

$$P((A \cap B)|F) = 0$$

$$\Rightarrow \quad P((A \cup B)|F) = P(A|F) + P(B|F)$$

Property 3

$$P(E'|F) = 1 - P(E|F)$$

From Property 1, we know that $P(S|F) = 1$

$\Rightarrow \quad P(E \cup E'|F) = 1 \quad$ since $S = E \cup E'$

$\Rightarrow \quad P(E|F) + P(E'|F) = 1 \quad$ since E and E′ are disjoint events

Thus, $\quad P(E'|F) = 1 - P(E|F)$

Multiplication Theorem on Probability

- Let E and F be two events associated with a sample spaceS.
- Conditional probability of event E given that F has occurred is denoted by P(E | F) and is givenby

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

- From this result, we canwrite

$$P(E \cap F) = P(F) \, . \, P(E|F) \qquad \dots (1)$$

Also, we know that

$$P(F|E) = \frac{P(F \cap E)}{P(E)}, P(E) \neq 0$$

or

$$P(F|E) = \frac{P(E \cap F)}{P(E)} \quad (\text{since } E \cap F = F \cap E)$$

Thus,

$$P(E \cap F) = P(E). \; P(F|E) \qquad \dots. (2)$$

Combining (1) and (2), we find that

$$P(E \cap F) = P(E) \; P(F|E)$$

$$= P(F) \; P(E|F) \text{ provided } P(E) \neq 0 \text{ and } P(F) \neq 0.$$

The above result is known as the **Multiplication rule of probability.**

- Example:
  - An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls areblack?
  - Solution:
    - Let E and F denote respectively the events that first and second ball drawn are black.
    - P(E) = P (black ball in first draw) =10/15
    - Given
      - First ball drawn is black, i.e., event E has occurred, now there are 9 black balls and five white balls left in theurn.
      - Therefore,theprobabilitythatthesecondballdrawnisblack,giventhatthe ball in the first draw is black, is nothing but the conditional probability of F given that E hasoccurred.
      - i.e. P(F | E) =9/14
    - By multiplication rule of probability, wehave

      P (E ∩ F) = P (E) P (F | E)

$$=10/15 \times 9/14 = 3/7$$

Note:

- Multiplication rule of probability for more than two events If E, F and G are three events of sample space, wehave

  $$P(E \cap F \cap G) = P(E)\, P(F|E)\, P(G|(E \cap F)) = P(E)\, P(F|E)\, P(G|EF)$$

- Similarly, the multiplication rule of probability can be extended for four or moreevents.

Independent Events

- If E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independentevents.
- Let E and F be two events associated with the same random experiment, then E and F are said to be independentif
  $P(E \cap F) = P(E).\, P(F)$

Remarks

- Two events E and F are said to be dependent if they are not independent, i.e. if $P(E \cap F) \neq P(E).\, P(F)$
- Sometimes there is a confusion between independent events and mutually exclusiveevents.
  - Term 'independent' is defined in terms of 'probability of events' whereas mutually exclusive is defined in term of events (subset of samplespace).
  - Mutuallyexclusiveeventsneverhaveanoutcomecommon,butindependentevents,may have commonoutcome.
  - Two independent events having nonzero probabilities of occurrence cannot be mutually exclusive, and conversely, i.e. two mutually exclusive events having nonzero probabilities of occurrence cannot be independent.

- Two experiments are said to be independent if for every pair of events E and F, where E is associated with the first experiment and F with the secondexperiment, the probability of the simultaneous occurrence of the events E and F when the two experiments are performed is the product of P(E) and P(F) calculated separately on the basis of two experiments, i.e. P (E ∩ F) = P (E). P (F)

- Three events A, B and C are said to be mutually independent, if P (A ∩ B) = P (A) P(B)
  P (A ∩ C) = P (A) P (C)
  P (B ∩ C) = P (B) P(C) and
  P (A ∩ B ∩ C) = P (A) P (B) P(C)
  If at least one of the above is not true for three given events, we say that the events are not independent.

- **Example**
  - A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent ? Solution:
    W.k.t the sample space is S = {1, 2, 3, 4, 5, 6} Now E = {3, 6}, F = {2, 4, 6}
    and E ∩ F = {6} Then
    - P(E) = 2/61/3
    - P(F) = 3/6 = 1/2
    - P(E ∩ F) =1/6

Clearly P (E ∩ F) = P (E). P (F). Hence E and F are independent events.

Bayes' Theorem: Description

- Also called as inverse probabilitytheorem
- Consider that there are two bags I andII.
  - Bag I contains 2 white and 3 redballs
  - Bag II contains 4 white and 5 redballs.
  - One ball is drawn at random from one of thebags.

- Probability of selecting any of the bags (i.e. 1/2) or probability of drawing a ball of a particular colour (say white) from a particular bag (say BagI).
- Probability that the ball drawn is of a particular colour, if we are given the bag from which the ball isdrawn.
- To find the probability that the ball drawn is from a particular bag (say Bag II), if the colour of the ball drawn is given we have to find the reverse probability of Bag II to be selected when an event occurred after it isknown.
- Famousmathematician,JohnBayes'solvedtheproblemoffindingreverseprobabilityby using conditionalprobability.
- Hence named as ‘Bayes theorem’ which was published posthumously in1763.

Definitions:

Partition of a sample space

- A set of events E1, E2, ..., En is said to represent a partition of the sample space Sif
  - $E_i \cap E_j = \varphi$, $i \neq j$, $i, j = 1, 2, 3, \ldots, n$
  - $E_1 \cup E_2 \cup \ldots \cup E_n = S$and
  - P (Ei) > 0 for all i = 1, 2,……,n.
- The events E1, E2, ..., En represent a partition of the sample space S if they are pairwise disjoint, exhaustive and have nonzeroprobabilities.

Theorem of total probability

- Let {E1, E2,...,En} be a partition of the sample spaceS,
- Suppose that each of the events E1, E2,...,En has nonzero probability ofoccurrence.
- Let A be any event associated with S,then

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + \ldots + P(E_n)\,P(A|E_n)$$

$$= \sum_{j=1}^{n} P(E_j)\,P(A|E_j)$$

Proof

Given that E1, E2,...,En is a partition of the sample space S. Therefore,

$$S = E_1 \cup E_2 \cup \ldots \cup E_n \qquad \ldots (1)$$

and $\qquad E_i \cap E_j = \phi,\ i \neq j,\ i, j = 1, 2, \ldots, n$

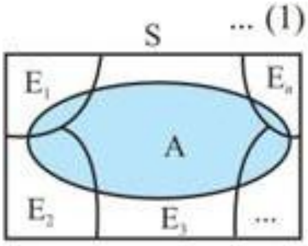


Fig 13.4

Now, we know that for any event A,

$$A = A \cap S$$
$$= A \cap (E_1 \cup E_2 \cup \ldots \cup E_n)$$
$$= (A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)$$

Also $A \cap E_i$ and $A \cap E_j$ are respectively the subsets of E_i and E_j. We know that E_i and E_j are disjoint, for $i \neq j$, therefore, $A \cap E_i$ and $A \cap E_j$ are also disjoint for all $i \neq j,\ i, j = 1, 2, \ldots, n$.

Thus, $P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup \ldots\ldots \cup (A \cap E_n)]$

$$= P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_n)$$

Now, by multiplication rule of probability, we have

$$P(A \cap E_i) = P(E_i)\, P(A|E_i) \text{ as } P(E_i) \neq 0 \; \forall i = 1, 2, \ldots, n$$

Therefore, $P(A) = P(E_1)\, P(A|E_1) + P(E_2)\, P(A|E_2) + \ldots + P(E_n) P(A|E_n)$

or $P(A) = \sum_{j=1}^{n} P(E_j) P(A|E_j)$

Bayes' Theorem: Proof

If E1, E2 ,..., En are n non empty events which constitute a partition of sample space S, i.e. E1, E2 ,..., En are pairwise disjoint and E1∪ E2∪ ... ∪ En = S and A is any event of nonzero probability,then

$$P(E_i|A) = \frac{P(E_i) P(A|E_i)}{\sum_{j=1}^{n} P(E_j) P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \ldots, n$$

Proof:

By formula of conditional probability, we know that

$$P(E_i|A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i) P(A|E_i)}{P(A)} \text{ (by multiplication rule of probability)}$$

$$= \frac{P(E_i) P(A|E_i)}{\sum_{j=1}^{n} P(E_j) P(A|E_j)} \text{ (by the result of theorem of total probability)}$$

Remark

The following terminology is generally used when Bayes' theorem is applied.

- The events E1, E2, ..., En are calledhypotheses.
- The probability P(Ei) is called the priori probability of the hypothesisEi
- The conditional probability P(Ei | A) is called a posteriori probability of the hypothesisEi.
- Alsocalledtheformulafortheprobabilityof"causes".SincetheEi'sareapartitionofthesa mple space S, one and only one of the events Ei occurs (i.e. one of the events E i

must occur and only one can occur). Hence, the above formula gives us the probability of a particular Ei, given that the event A hasoccurred.

Random Variables and its Probability Distributions

Inmostoftherandomexperimentsand Samplespace,wewerenotonlyinterestedinthepar ticular outcome that occurs but rather in some number associated with that outcomes as shown in following examples/experiments.

- Experiments
  - In tossing two dice, we may be interested in the sum of the numbers on the twodice.
  - In tossing a coin 50 times, we may want the number of headsobtained.
  - In the experiment of taking out four articles (one after the other) at random from a lot of 20 articles in which 6 are defective, we want to know the number of defectives in the sample of four and not in the particular sequence of defective and non-defectivearticles.

- In all the aboveexperiments,
  - We have a rule which assigns to each outcome of the experiment a single realnumber.
  - Thissinglerealnumbermayvarywithdifferentoutcomesoftheexperiment.Hen ce,itis a variable.
  - Also its value depends upon the outcome of a random experiment and, hence, is called randomvariable.
  - A random variable is usually denoted byX.
  - A random variable can take any real value, therefore, its co-domain is the set of real numbers. Hence, a random variable can be defined asfollows
    - A random variable is a real valued function whose domain is the sample space of a randomexperiment.
    - Eg: Consider the experiment of tossing a coin two times insuccession
    - Sample space of the experiment is S = {HH, HT, TH,TT}.
    - If X denotes the number of heads obtained, then X is a random variable and for each outcome, its value is as given below:
      - X(HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) =0
    - Let Y denote the number of heads minus the number of tails for each outcome of the above sample spaceS.
      - Y (HH) = 2, Y (HT) = 0, Y (TH) = 0, Y (TT) = –2.
    - Hence, X and Y are two different random variables defined on the same sample spaceS.

Note: More than one random variables can be defined on the same samplespace.

Probability distribution of a random variable

- Descriptiongivingthevaluesoftherandomvariablealongwiththecorrespondingprobabilities is called the probability distribution of the random variableX.
- In general, the probability distribution of a random variable X is defined asfollows:
  - The probability distribution of a random variable X is the system ofnumbers

X	:	x_1	x_2	...	x_n
P(X)	:	p_1	p_2	...	p_n

where, $p_i > 0, \ \sum_{i=1}^{n} p_i = 1, i = 1, 2, ..., n$

The real numbers $x_1, x_2, ..., x_n$ are the possible values of the random variable X and p_i ($i = 1,2,..., n$) is the probability of the random variable X taking the value x_i i.e., $P(X = x_i) = p_i$

- Also for all possible values of the random variable X, all elements of the sample space are covered. Hence, the sum of all the probabilities in a probability distribution must beone.
- If xi is one of the possible values of a random variable X, the statement X=xiistrueonlyatsomepoint(s)ofthesamplespace.Hence,theprobabilitythatXtakesvalue xi is always nonzero, i.e. P(X = xi)≠ 0

Mean of a random variable

- Mean is a measure of location or central tendency in the sense that it roughly locates a middle or average value of the randomvariable.
- Let X be a random variable whose possible values x1, x2, x3, ..., xn occur with probabilities p1,p2, $\sum_{i=1}^{n} x_i p_i$

p3,...,pn,respectively.ThemeanofX,denotedbyμ,isthenumber i.e. the mean of X is the weighted average of the possible values of X, each value being weighted by its probability with which itoccurs.

- The mean of a random variable X is also called the expectation of X, denoted byE(

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \dots + x_n p_n.$$

Thus

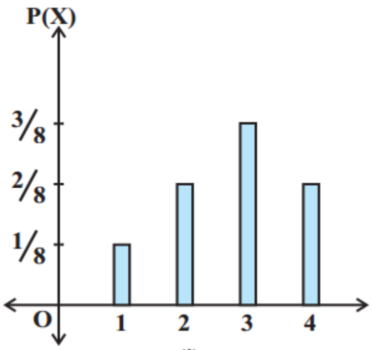
- ThemeanorexpectationofarandomvariableXisthesumoftheproductsofallpossiblevalues of X by their respectiveprobabilities.

Variance of a random variable

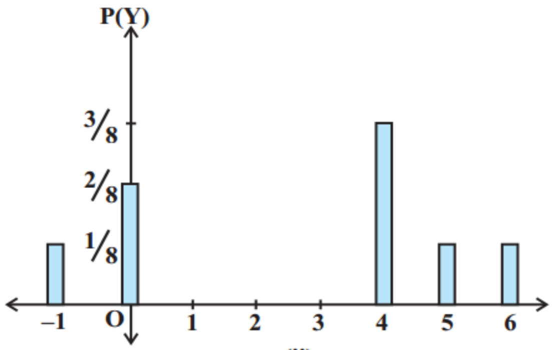
- The mean of a random variable does not give us information about the variability in the values of the randomvariable.
- If the variance is small, then the values of the random variable are close to the mean. Also randomvariableswithdifferentprobabilitydistributionscanhaveequalmeans,asshowninthe following distributions of X andY

X	1	2	3	4
P(X)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$

The variables X and Y are different, however their means are same.
The diagrammatic representation of these distributions are shown below:



(i)



(ii)

- Let X be a random variable whose possible values x1, x2,...,xn occur with

Let μ = E (X) be the mean of X. The variance of X, denoted by Var (X) or $\sigma_x^{\ 2}$ is defined as

$$\sigma_x^{\ 2} = \mathrm{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)$$

or equivalently $\qquad \sigma_x^{\ 2} = E(X - \mu)^2$

$$\sigma_x = \sqrt{\mathrm{Var}(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)}$$

probabilities p(x1), p(x2),..., p(xn)respectively.

The non- negative number is called the standard deviation of the random variable

Another formula to find the variance of a random variable. We know that,

$$\mathrm{Var}\,(X) = \sum_{i=1}^{n} (x_i - \mu)^2 \, p(x_i)$$

$$= \sum_{i=1}^{n} (x_i^{\ 2} + \mu^2 - 2\mu\, x_i)\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i) + \sum_{i=1}^{n} \mu^2\, p(x_i) - \sum_{i=1}^{n} 2\mu x_i\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i) + \mu^2 \sum_{i=1}^{n} p(x_i) - 2\mu \sum_{i=1}^{n} x_i\, p(x_i)$$

$$= \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i) + \mu^2 - 2\mu^2 \left[\text{since} \sum_{i=1}^{n} p\,(x_i) = 1 \,\text{and}\, \mu = \sum_{i=1}^{n} x_i\, p(x_i) \right]$$

$$= \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i) - \mu^2$$

or $$\mathrm{Var}\,(X) = \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i) - \left(\sum_{i=1}^{n} x_i\, p(x_i) \right)^2$$

or $$\mathrm{Var}\,(X) = E(X^2) - [E(X)]^2, \text{ where } E(X^2) = \sum_{i=1}^{n} x_i^{\ 2}\, p(x_i)$$

Bernoulli Trials and Binomial Distribution

Bernoulli trials

- Theoutcomeofanytrialisindependentoftheoutcomeofanyothertrial.Ineachof suchtrials, the probability of success or failure remains constant. Such independent trials which have only two outcomes usually referred as 'success' or 'failure' are called Bernoullitrials.
- Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:
  - There should be a finite number oftrials.
  - The trials should beindependent.
  - Each trial has exactly two outcomes: success orfailure.
  - The probability of success remains the same in eachtrial.
  - Example: 30 Six balls are drawn successively from an urn containing 7 red and 9 black balls. Tell whether or not the trials of drawing balls are Bernoulli trials when after each draw the ball drawn is (i) replaced (ii) not replaced in theurn.

    Solution

    (i) The number of trials is finite. When the drawing is done with replacement, the probability of success (say, red ball) is p = 7/16 which is same for all six trials (draws). Hence, the drawing of balls with replacements are Bernoullitrials.

    (ii) When the drawing is done without replacement, the probability of success (i.e., red ball)infirsttrialis7/16,in2ndtrialis6/15ifthefirstballdrawnisredor7/15ifthefirst ball drawn is black and so on. Clearly, the probability of success is not same for all trials, hence the trials are not Bernoullitrials

Binomial distribution

TheprobabilitydistributionofnumberofsuccessesinanexperimentconsistingofnBernoullitrials may be obtained by the binomial expansion of $(q + p)^n$. Hence, this distribution of number of successes X can be written as

X	0	1	2	...	x	...	n
P (X)	${}^nC_0\, q^n$	${}^nC_1\, q^{n-1}p^1$	${}^nC_2\, q^{n-2}p^2$		${}^nC_x\, q^{n-x}p^x$		${}^nC_n\, p^n$

The above probability distribution is known as binomial distribution with parameters n and p, because for given values of n and p, we can find the complete probability distribution.

The probability of x successes P(X = x) is also denoted by P(x) and is given by

$$P(x) = {}^nC_x q^{n-x} p^x, \quad x = 0, 1, \ldots, n. \ (q = 1 - p)$$

ThisP(x)iscalledtheprobabilityfunctionofthebinomialdistribution.Abinomialdistributionwith n-Bernoulli trials and probability of success in each trial as p, is denoted by B (n,p).

MCQS

1. Two Persons A and B appear in an interview for two vacancies. If the Probability of their selection are $\frac{1}{4}$ and $\frac{1}{6}$ respectively. Then probability that none of them is selected is:

(a) $\frac{1}{24}$ (b) $\frac{5}{12}$ (c) $\frac{5}{8}$ (d) $\frac{19}{12}$

2. Three letters are sent to different persons and addresses on the three envelopes are written at random. The Probability that the letters go into the right envelopes is:

(a) $\frac{1}{27}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) None of these

3. A bag contains n coupons marked 1,2,3, ……….n. if two coupons are drawn , then the chance that the difference of the coupons exceeds m (less than n-1) is:

(a) $\frac{(n-m)(n+m-1)}{n(n-1)}$ (b) $\frac{(n-m)(n-m-1)}{n(n-1)}$ (c) $\frac{(n+m)(n+m-1)}{n(n-1)}$ (d) None of these

4. Probabilities that a plant will live is $\frac{3}{4}$ and the probability that another plant lives is $\frac{1}{3}$. The probability that only one of them lives is:

(a) $\frac{7}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) None of these

5. A sample space consists of three mutually independent and equally likely events. The Probability of happening of each one of them is equal to:

(a) 0 (b) $\frac{1}{3}$ (c) 1 (d) None of these

6. For any two independent event E_1 and E_2 in a space S. P is

(a) $\leq\frac{1}{4}$ (b) $>\frac{1}{4}$ (c) $\geq\frac{1}{2}$ (d) $>\frac{1}{2}$

7. One bag contains 6 blue and 5 greens balls and another bag contains 7 blue and 4 green balls. Two balls are drawn ,one from each bag. The Probability of both being blue is:

(a) $\frac{42}{121}$ (b) $\frac{20}{121}$ (c) $\frac{3}{11}$ (d) $\frac{2}{11}$

8. For any two events A and B , P(A∩ B is:

(a) Less than P(A)+P(B)-1 (b) Greater than P(A)+P(B)

(c) Equal to P(A)+P(B) – P(A∪B) (d) P(A)+P(B) + P(A∪B)

9. Tickets are numbered 1 to 100 . They are well-shuffled and a ticket is drawn at random. Probability that the drawn ticket has a number 5 or multiple of 5 is:

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{1}{25}$ (d) $\frac{1}{2}$

10. There are two boxes. One box contains 3 white balls and 2 black balls. The other

box contains 7 yellow balls and 3 black balls. If a box is selected at random and from it , a ball is drawn, the probability that the ball is back is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$ (c)$\frac{3}{20}$ (d) $\frac{1}{3}$

11. If the probability that A and B will die within a year are p and q respectively .then the probability that only one of them will be alive at the year is:

(a) p+q (b) p+q$-2pq$ (c) p+q+pq (d) p+q-pq

12. The chance of throwing an ace in the first out of two successive throws with an ordinary dice is:

(a) $\frac{1}{6}$ (b) $\frac{5}{36}$ (c)$\frac{1}{36}$ (d) $\frac{25}{36}$

13. A and B are two events such that P(A)>0 , P(B) ≠ 1 then $P\left(\acute{A}/\acute{B}\right)$ is equal to:

(a) 1 - P(A/B) (b) 1- $P\left(\acute{A}/B\right)$ (c) $\frac{1-P(A\cup B)}{P(\acute{B})}$ (d) $P\left(\acute{A}\right)/P\left(\acute{B}\right)$

14. A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, Then P(E/F)=

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c)$\frac{1}{2}$ (d) $\frac{1}{8}$

15. if from each of the three boxes containing 3 white and 1 black , 2 white and 2 black, 1 white and 3 black balls , one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is:

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c)$\frac{1}{32}$ (d) $\frac{3}{16}$

16. A fair coin is tossed repeatedly .if tail appears on first four tosses, then the probability of head appearing on fifth toss equals:

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$ (c)$\frac{31}{32}$ (d) $\frac{1}{5}$

17. if$\acute{E}$ and $\acute{F}$ are complementary events of events E and F respectively and 0<P(F)<1 then :

(a) P(E/F)+ P($\acute{E}/F = 1$ or P(E/$\acute{F} + P\left(\acute{E}/\acute{F}\right)$ =1

(b) P(E/F)+P(E/$\acute{F} = 1$

(c) P($\acute{E}/F$ +P(E/$\acute{F} = 1$

(d) None of these

18. if E and F are events with P(E)≤P(F) and P(E∩ $F > 0$ then:

(a) occurrence of E⇒ occurrence of F

(b) occurrence of F ⇒ occurrence of E

(c) nonoccurrence of E⇒non occurrence of F

(d) None of the above implication holds

19. For any two events A and B P(A∪ $B \cap P\left(\acute{A} \cap \acute{B}\right)$ is:

(a) ≤ $\frac{1}{3}$ (b) >$\frac{1}{3}$ (c) >$\frac{1}{2}$ (d) ≥ $\frac{1}{2}$

20. A card is drawn at random from a pack of 100 cards numbered from 1 to 100. The Probability of drawing a number which is a square is:

(a) 1/5 (b) 2/5 (c) 1/10 (d) None of these

21. From a well shuffled pack of playing cards, two cards drawn one by one with replacement. The probability that both aces is:

(a) 2/13 (b) 1/51 (c) 1/221 (d) None of these

22. In tossing 10 coins, the probability of getting exactly 5 heads is :

(a) $\frac{9}{128}$ (b) $\frac{63}{256}$ (c)$\frac{1}{2}$ (d) $\frac{193}{256}$

23. A binomial probability distribution is symmetrical if p the probability of success in a single trial is:

(a) less than $\frac{1}{2}$ (b) greater than $\frac{1}{2}$ (c) equal to $\frac{1}{2}$ (d) less than q where q=1-p

24. Let X and Y be two random variables. The relationship E(XY)=E(X)E(Y) holds:

(a) ALWAYS (b) if E(X+Y)=E(X)+E(Y) is true

(c) if X and Y are independent (d) if X can be obtained from Y by a linear transformation

25.if X denotes the number of sixes in four consecutive throws of a dice, then P(X=4) is:

(a) $\frac{1}{1296}$ (b) $\frac{4}{6}$ (c) 1 (d) $\frac{1295}{1296}$

26.if the mean of the binomial distribution is 20 and standard deviation is 4, then the number of events is:

(a) 50 (b) 25 (c) 100 (d) 80

27. if mean of a bacterial distribution is 3 and its variance is $\frac{3}{2}$ then number of trials is:

(a) 6 (b) 2 (c) 12 (d) None of these

ANSWERS

1	**c**	2	**b**	3	**b**	4	**a**	5	**b**	6	**a**	7	**a**	8	**c**	9	**b**
10	**a**	11	**b**	12	**b**	13	**c**	14	**a**	15	**a**	16	**a**	17	**a**	18	**d**
19	**a**	20	**c**	21	**c**	22	**b**	23	**c**	24	**c**	25	**a**	26	**c**	27	**a**