Relations and Functions

Definitions:

Let A and B be two non-empty sets, then a function f from set A to set B is a rule which associates each element of A to a unique element of B.

- **Relation**

  If $(a, b) \in R$, we say that a is related to b under the relation R and we write as a R b

- **Function**

  It is represented as $f: A \to B$ and function is also called mapping.

- **Real Function**

  $f: A \to B$ is called a real function, if A and B are subsets of R.

- **Domain and Codomain of a Real Function**

  Domain and codomain of a function f is a set of all real numbers x for which f(x) is a real number. Here, set A is domain and set B is codomain.

- **Range of a real function**

  f is a set of values f(x) which it attains on the points of its domain

Types of Relations

- A relation R in a set A is called **Empty relation**, if no element of A is related to any element of

  A, i.e., $R = \varphi \subset A \times A$.
- A relation R in a set A is called **Universal relation**, if each element of A is related to every element of A, i.e., $R = A \times A$.
- Both the empty relation and the universal relation are sometimes called **Trivial Relations**
- A relation R in a set A is called
  - **Reflexive**
    - if $(a, a) \in R$, for every $a \in A$,
  - **Symmetric**
    - If $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
  - **Transitive**
    - If $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

- A relation R in a set A is said to be an **equivalence relation** if R is reflexive, symmetric and transitive

- The set E of all even integers and the set O of all odd integers are subsets of Z satisfying following conditions:
  - All elements of E are related to each other and all elements of O are related to each other.
  - No element of E is related to any element of O and vice-versa.
  - E and O are disjoint and $Z = E \cup O$.
  - The subset E is called the equivalence class containing zero, Denoted by [0].
  - O is the equivalence class containing 1 and is denoted by [1].

- **Note**
  - $[0] \neq [1]$
  - $[0] = [2R]$
  - $[1] = [2R + 1]$, $r \in Z$.

- Given an arbitrary equivalence relation R in an arbitrary set X, R divides X into mutually disjoint subsets Ai called partitions or subdivisions of X satisfying:
  - All elements of Ai are related to each other, for all i.
  - No element of Ai is related to any element of Aj, $i \neq j$.
  - $\bigcup A_j = X$ and $A_i \cap A_j = \varphi$, $i \neq j$.
- The subsets A_i are called equivalence classes.

Note:

- Two ways of representing a relation
  - Roaster method
  - Set builder method
- If $(a, b) \in R$, we say that a is related to b and we denote it as $\boldsymbol{a\,R\,b}$.

Classification of Real Functions

Real functions are generally classified under two categories algebraic functions and transcendental functions.

1. Algebraic Functions

Some algebraic functions are given below
(i) Polynomial Functions If a function y = f(x) is given by

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = \sum_{i=0}^{n} a_i x^{n-i}$$

where, a_0, a_1, a_2,…, a_n are real numbers and n is any non -negative integer, then f (x) is called a polynomial function in x.

If $a_0 \neq 0$, then the degree of the polynomial f(x) is n. The domain of a polynomial function is the set of real number R.

e.g., $y = f(x) = 3x^5 - 4x^2 - 2x + 1$

is a polynomial of degree 5.

(ii) Rational Functions If a function y = f(x) is given by f(x) = φ(x) / Ψ(x)

where, φ(x) and Ψ(x) are polynomial functions, then f(x) is called rational function in x.

(**iii) Irrational Functions**The algebraic functions containing one or more terms having nonintegral rational power x are called irrational functions.
e.g., $y = f(x) = 2\sqrt{x} - \sqrt[3]{x} + 6$

2. Transcendental Function

A. function, which is not algebraic, is called a transcendental function. Trigonometric, Inverse trigonometric, Exponential, Logarithmic, etc are transcendental functions.

Explicit and Implicit Functions

(i) Explicit Functions A function is said to be an explicit function, if it is expressed in the form y = f(x).

(ii) Implicit Functions A function is said to be an implicit function, if it is expressed in the form f(x, y) = C, where C is constant.
e.g., sin (x + y) – cos (x + y) = 2

Intervals of a Function

(i) The set of real numbers x, such that a ≤ x ≤ b is called a closed interval and denoted by [a, b] i.e., {x: x ∈ R, a ≤ x ≤ b}.

(ii) Set of real number x, such that a < x < b is called open interval and is denoted by (a, b) i.e., {x: x ∈ R, a < x < b}

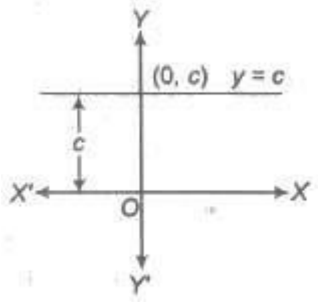
(iii) Intervals [a,b) = {x: x ∈ R, a ≤ x ≤ b} and (a, b] = {x: x ≠ R, a < x ≤ b} are called semiopen and semi-closed intervals.

Graph of Real Functions

1. Constant Function Let c be a fixed real number.

The function that associates to each real number x, this fixed number c is called a constant function i.e., y = f{x) = c for all x ∈ R.

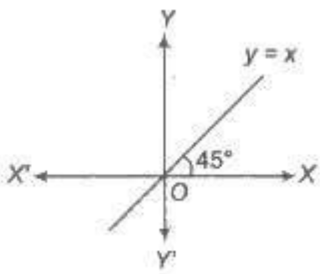
Domain of f{x) = R
Range of f{x) = {c}



2. Identity Function

The function that associates to each real number x for the same number x, is called the identity function. i.e., y = f(x) = x, ∀ x ∈

R. Domain of f(x) = R
Range f(x) = R

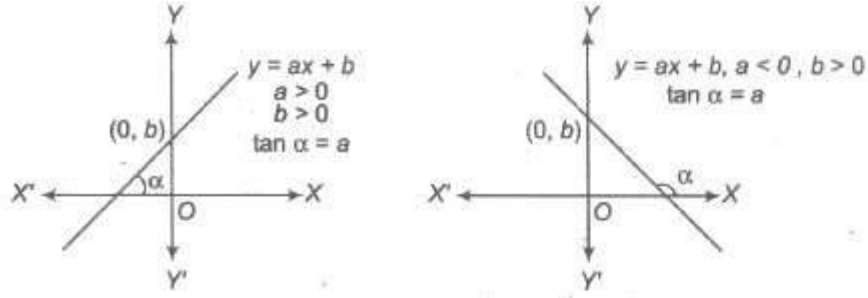


3. Linear Function

If a and b be fixed real numbers, then the linear function is defmed as y = f(x) = ax + b, where a and b are constants.

Domain of f(x) = R
Range of f(x) = R

The graph of a linear function is given in the following diagram, which is a straight line with slope a.



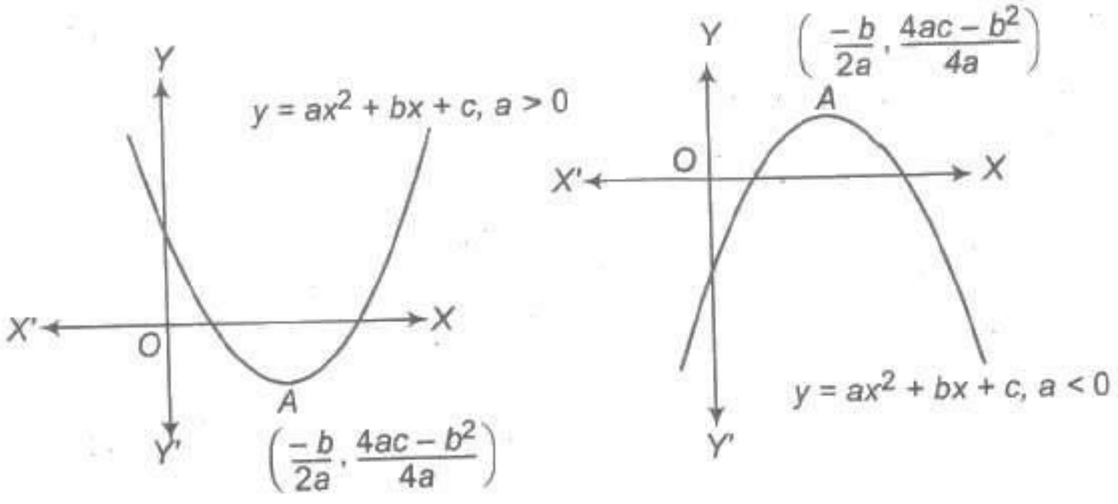
4. Quadratic Function

If a, b and c are fixed real numbers, then the quadratic function is expressed as $y = f(x) = ax^2 + bx + c, a \neq 0 \Rightarrow y = a (x + b / 2a)^2 + 4ac - b^2 / 4a$

which is equation of a parabola in downward, if $a < 0$ and upward, if $a > 0$ and vertex at $(-b / 2a, 4ac - b^2 / 4a)$.

Domain of f(x) = R

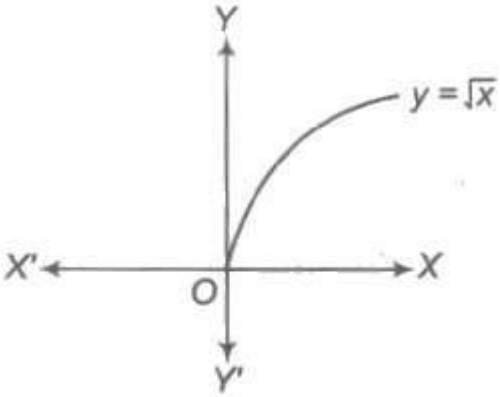
Range of f(x) is $[-\infty, 4ac - b^2 / 4a]$, if $a < 0$ and $[4ac - b^2 / 4a, \infty]$, if $a > 0$ 5. Square Root Function Square root function is defined by $y = F(x) = \sqrt{x}, x \geq 0$.



5. Square Root Function

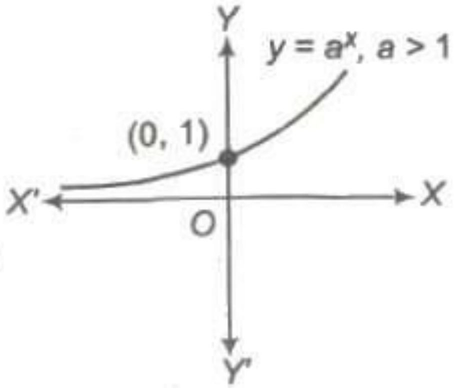
Square root function is defined by $y = F(x) = \sqrt{x}, x \geq 0$.

Domain of $f(x) = [0, \infty)$
Range of $f(x) = [0, \infty)$



6. Exponential Function

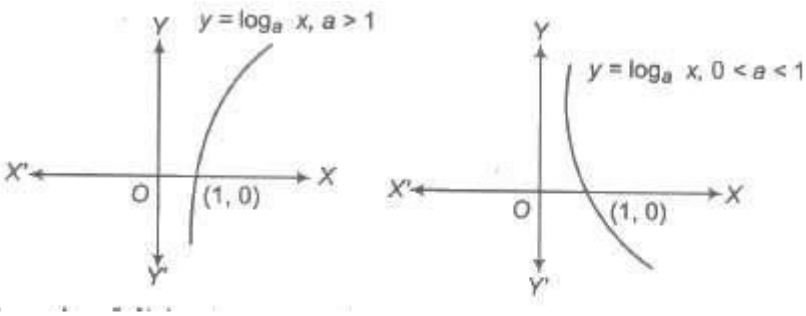
Exponential function is given by $y = f(x) = a^x$, where $a > 0$, $a \neq 1$.



7. Logarithmic Function

A logarithmic function may be given by $y = f(x) = \log_a x$, where $a > 0$, $a \neq 1$ and $x > 0$.

The graph of the function is as shown below. which is increasing, if $a > 1$ and decreasing, if $0 < a < 1$.

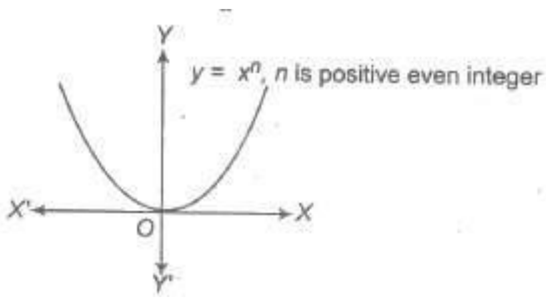


Domain of $f(x) = (0, \infty)$
Range of $f(x) = R$

8. Power Function

The power function is given by $y = f(x) = x^n$, $n \in I, n \neq 1, 0$. The domain and range of the graph $y = f(x)$, is depend on n.

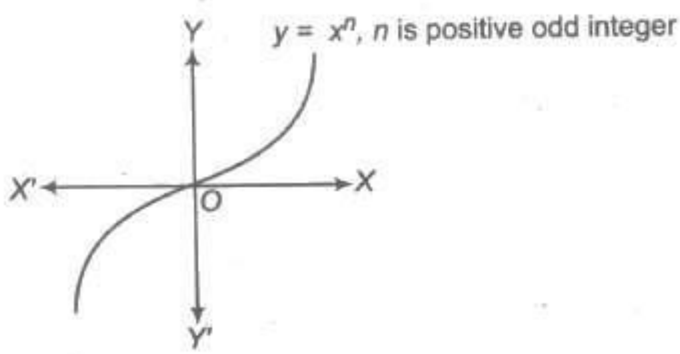
(a) If n is positive even integer.



i.e., $f(x) = x^2, x^4$,….

Domain of $f(x) = R$
Range of $f(x) = [0, \infty)$

(b) If n is positive odd integer.

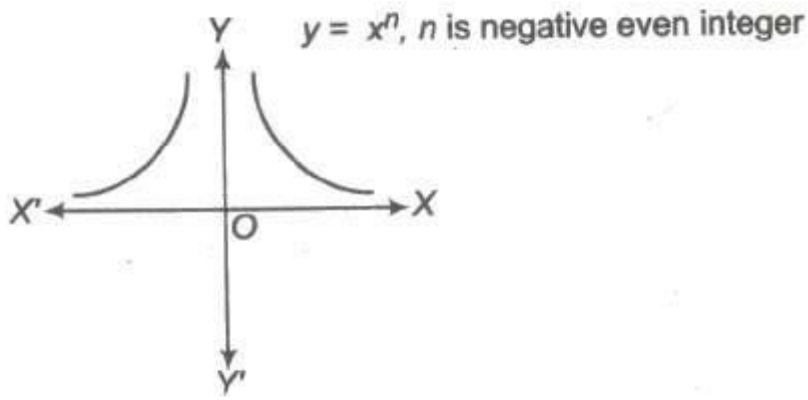


i.e., $f(x) = x^3, x^5$,….

Domain of $f(x) = R$
Range of $f(x) = R$

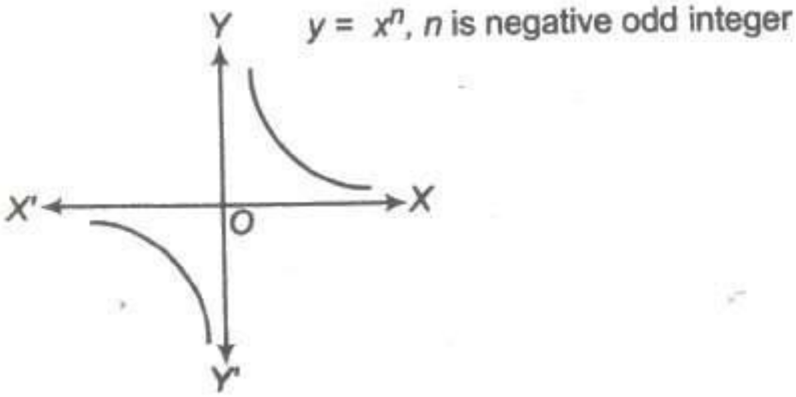
(c) If n is negative even integer.

i.e., $f(x) = x^{-2}, x^{-4}$,….



Domain of $f(x) = R – \{0\}$
Range of $f(x) = (0, \infty)$

(d) If n is negative odd integer.



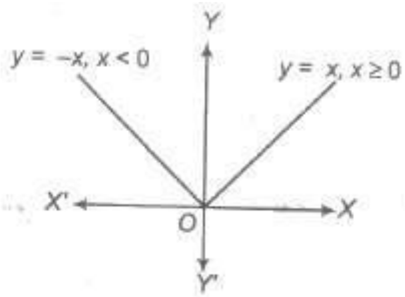
i.e., $f(x) = x^{-1}, x^{-3}, \ldots$

Domain of $f(x) = R - \{0\}$
Range of $f(x) = R - \{0\}$

9. Modulus Function (Absolute Value Function)

Modulus function is given by $y = f(x) = |x|$, where $|x|$ denotes the absolute value of x, that is

$|x| = \{x, \text{ if } x \geq 0, -x, \text{ if } x < 0$

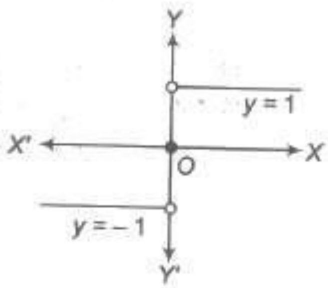


Domain of $f(x) = R$
Range of f(x) = [0, &infi;)

10. **Signum Function**

Signum function is defined as follows

$$y = f(x) = \begin{cases} \frac{|x|}{x}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases} \text{ or } \begin{cases} \frac{x}{|x|}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases}$$



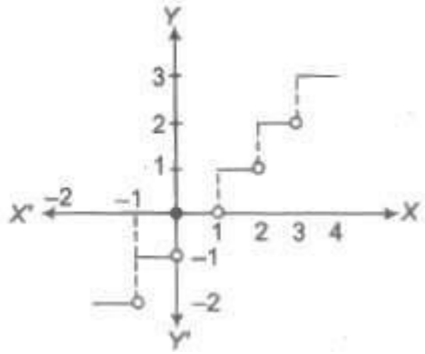
Symbolically, signum function is denoted by sgn (x).

Thus, $y = f(x) = \text{sgn}(x)$

where, sgn $(x) = \begin{cases} \frac{|x|}{x}, \text{ if } x \neq 0 \\ 0, \text{ if } x = 0 \end{cases} = \begin{cases} -1, \text{ if } x < 0 \\ 0, \text{ if } x = 0 \\ 1, \text{ if } x > 0 \end{cases}$

Domain of $f(x) = R$
Range of $f(x) = \{-1, 0, 1\}$

11. Greatest Integer Function



The greatest integer function is defined as y = f(x) = [x]

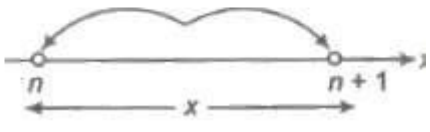
where, [x] represents the greatest integer less than or equal to x. i.e., for any integer n, [x] = n, if $n \leq x < n + 1$ Domain of f(x) = R Range of f(x) = I

Properties of Greatest Integer Function

(i) [x + n] = n + [x], $n \in I$
(ii) x = [x] + {x}, {x} denotes the fractional part of x.
(iii) [- x] = – [x], $-x \in I$
(iv) [- x] = – [x] – 1, $x \in I$
(v) $[x] \geq n \Rightarrow x \geq n, n \in I$
(vi) $[x] > n \Rightarrow x \Rightarrow n+1, n \in I$
(vii) $[x] \leq n \Rightarrow x < n + 1, n \in I$
(viii) $[x] < n \Rightarrow x < n, n \in I$
(ix) [x + y] = [x] + [y + x – [x}] for all $x, y \in R$
(x) $[x + y] \geq [x] + [y]$
(xi) [x] + [x + 1 / n] + [x + 2 / n] +…+ [x + n – 1 / n] = [nx], $n \in N$

12. Least Integer Function

The least integer function which is greater than or equal to x and it is denoted by (x). Thus, (3.578) = 4, (0.87) = 1, (4) = 4, (- 8.239) = – 8, (- 0.7) = 0



In general, if n is an integer and x is any real number between n and (n + 1).
i.e., $n < x \leq n + 1$, then (x) = n + 1

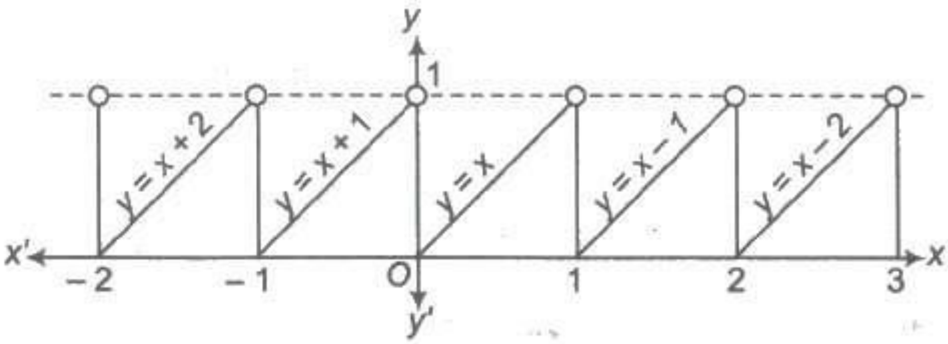
∴f(x) = (x)

Domain of f = R
Range of f= [x] + 1

13. Fractional Part Function

It is denoted as f(x) = {x} and defined as

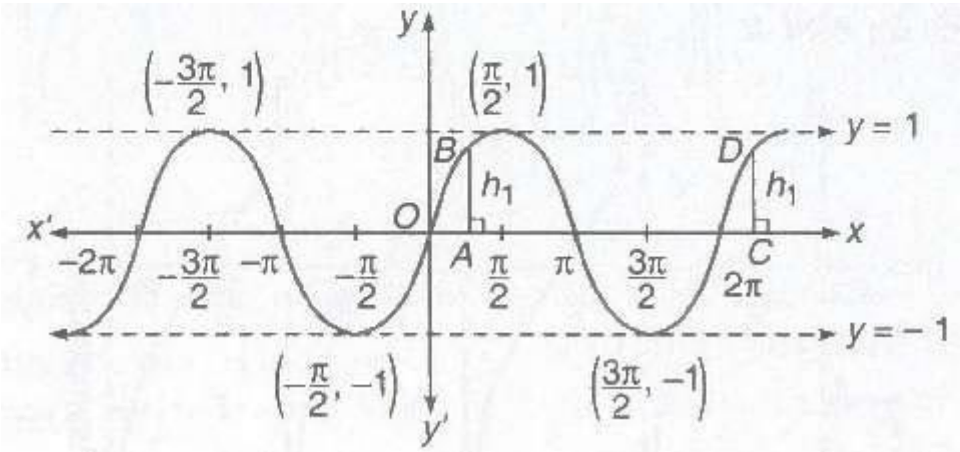
(i) {x} = f, if x = n + f, where n ∈ I and $0 \le f < 1$
(ii) {x} = x – [x]



i.e., {O.7} = 0.7, {3} = 0, { – 3.6} = 0.4
(iii) {x} = x, if $0 \le x \le 1$
(iv) {x} = 0, if x ∈ I
(v) { – x} = 1 – {x}, if x ≠ I

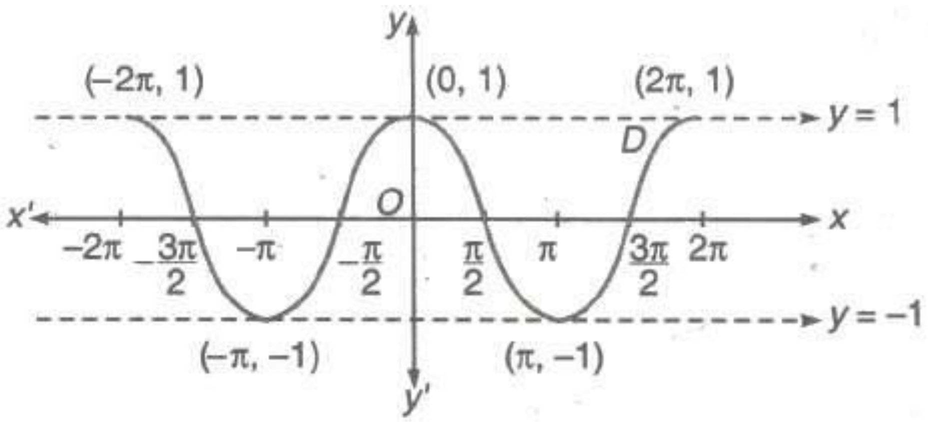
Graph of Trigonometric Functions

1. Graph of sin x



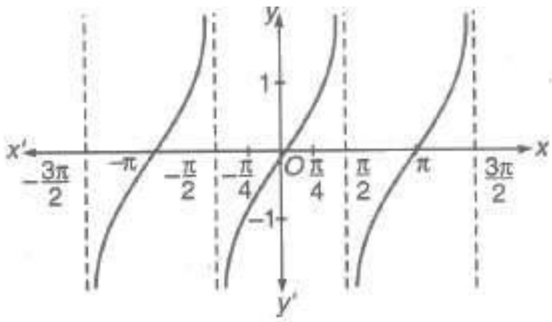
(i) Domain = R
(ii) Range = [-1,1]
(iii) Period = 2π

2. Graph of cos x



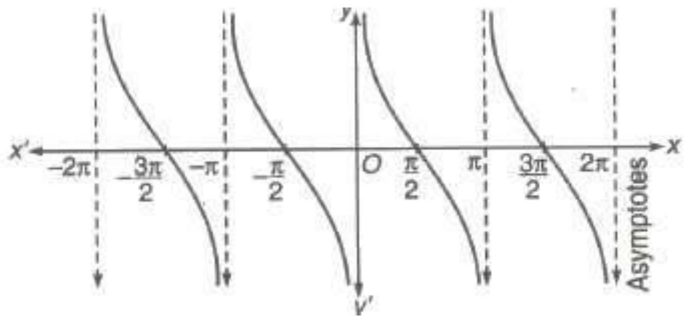
(i) Domain = R
(ii) Range = [-1,1]
(iii) Period = 2π

3.Graph of tan x



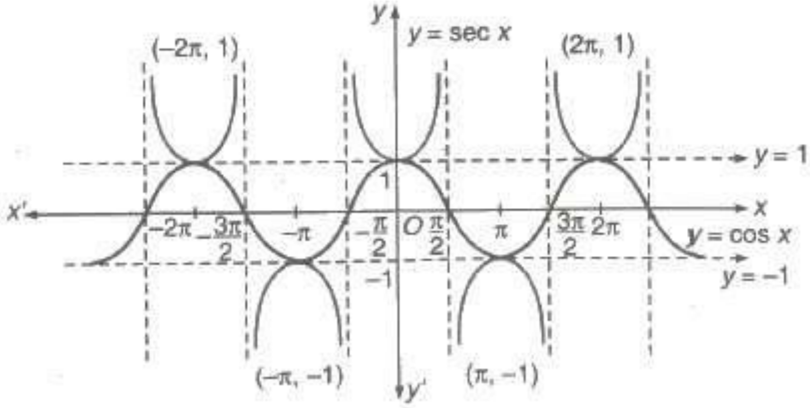
(i) Domain = R ~ (2n + 1) π / 2, n ∈I
(ii) Range = [- &infi;, &infi;]
(iii) Period = π

4. Graph of cot x



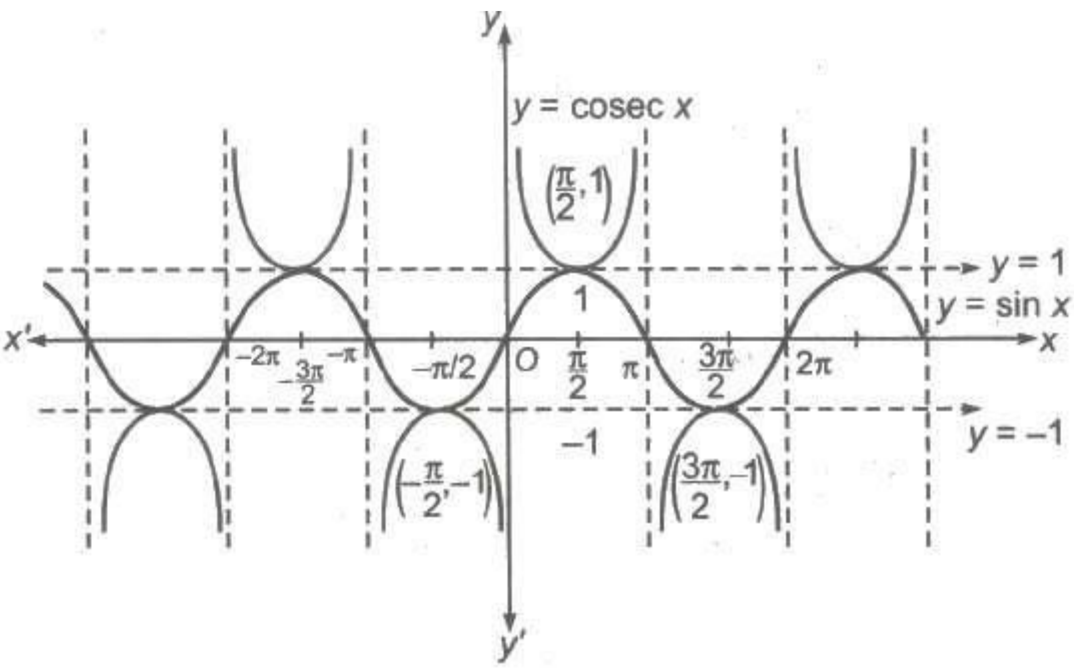
(i) Domain = R ~ nπ, n ∈I
(ii) Range = [- &infi;, &infi;]
(iii) Period = π

5. Graph of sec x



(i) Domain = R ~ (2n + 1) π / 2, n ∈I
(ii) Range = [- &infi;, 1] ∪ [1, &infi;)
(iii) Period = 2π

6. Graph of cosec x



(i) Domain = R ~ nπ, n ∈I
(ii) Range = [- &infi;, – 1] ∪ [1, &infi;)
(iii) Period = 2π

Operations on Real Functions

Let f: x → R and g : X → R be two real functions, then

(i) Sum The sum of the functions f and g is defined as f + g : X → R such that (f + g) (x) = f(x) + g(x).
(ii) Product The product of the functions f and g is defined as fg : X → R, such that (fg) (x) = f(x) g(x) Clearly, f + g and fg are defined only, if f and g have the same domain. In case, the domain of f and g are different. Then, Domain of f + g or fg = Domain of f ∩ Domain of g.
(iii) Multiplication by a Number Let f : X → R be a function and let e be a real number . Then, we define cf: X → R, such that (cf) (x) = cf (x), ∀ x ∈ X.
(iv)Composition (Function of Function) Let f : A → B and g : B → C be two functions. We define gof : A → C, such that got (c) = g(f(x)), ∀ x ∈ A
Alternate There exists Y ∈ B, such that if f(x) = y and g(y) = z, then got (x) = z

Periodic Functions

A function f(x) is said to be a periodic function of x, provided there exists a real number T > 0, such that F(T + x) = f(x), ∀ x ∈ R

The smallest positive real number T, satisfying the above condition is known as the period or the fundamental period of f(x) ..

Testing the Periodicity of a Function

(i) Put f(T + x) = f(x) and solve this equation to find the positive values of T independent of x.
(ii) If no positive value of T independent of x is obtained, then f(x) is a non-periodic function.
(iii) If positive val~esofT independent of x are obtained, then f(x) is a periodic function and the least positive value of T is the period of the function f(x).

Important Points to be Remembered

(i) Constant function is periodic with no fundamental period.
(ii) If f(x) is periodic with period T, then 1 / f(x) and. √f(x) are also periodic with f(x) same period T.
{iii} If f(x) is periodic with period T_1 and g(x) is periodic with period T_2, then f(x) + g(x) is periodic with period equal to LCM of T_1 and T_2, provided there is no positive k, such that f(k + x) = g(x) and g(k + x) = f(x).
(iv) If f(x) is periodic with period T, then kf (ax + b) is periodic with period T / |a|' where a, b ,k∈ R and a, k ≠ 0.
(v) sin x, cos x, sec x and cosec x are periodic functions with period 2π.
(vi) tan x and cot x are periodic functions with period π.
(vii) |sin x|, |cos x|, |tan x|, |cot x|, |sec x| and |cosec x| are periodic functions with period π.
(viii) $\sin^n$ x, $\cos^n$ x, $\sec^n$ x and cosec^nx are periodic functions with period 2π when n is odd, or π when n is even .
(ix) $\tan^n$ x and $\cot^n$x are periodic functions with period π.
(x) |sin x| + |cos x|, |tan x| + |cot x| and |sec x| + |cosec x| are periodic with period π / 2.

Even and Odd Functions

Even Functions A real function f(x) is an even function, if f(-x) = f(x).
Odd Functions A real function f(x) is an odd function, if f(-x) = – f(x).

Properties of Even and Odd Functions

(i) Even function ± Even function = Even function.
(ii) Odd function ± Odd function = Odd function.
(iii) Even function * Odd function = Odd function.
(iv) Even function * Even function = Even function.
(v) Odd function * Odd function = Even function.
(vi) gof or fog is even, if anyone of f and g or both are even.
(vii) gof or fog is odd, if both of f and g are odd.
(viii) If f(x) is an even function, then d / dx f(x) or ∫ f(x) dx is odd and if dx ..f(x) is an odd function, then d / dx f(x) or ∫ f(x) dx is even.
(ix) The graph of an even function is symmetrical about Y-axis.
(x) The graph of an odd function is symmetrical about origin or symmetrical in opposite quadrants.
(xi) An even function can never be one-one, however an odd function mayor may not be oneone.

Different Types of Functions (Mappings)

1. One-One and Many-One Function

The mapping f: A → B is a called one-one function, if different elements in A have different images in B. Such a mapping is known as injective function or an injection.

Methods to Test One-One

(i) Analytically If x_1, $x_2 \in$ A, then $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ or equivalently $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
(ii) Graphically If any .line parallel to x-axis cuts the graph of the function atmost at one point, then the function is one-one.
(iii) Monotonically Any function, which is entirely increasing or decreasing in whole domain, then f(x) is one-one.
Number of One-One Functions Let f : A → B be a function, such that A and B are finite sets having m and n elements respectively, (where, n > m).
The number of one-one functions $n(n - 1)(n - 2) \ldots(n - m + 1) = \{ {}^nP_m, n \geq m, 0, n < m$
The function f : A → B is called many – one function, if two or more than two different elements in A have the same image in B.

2. Onto (Surjective) and Into Function

If the function f: A → B is such that each element in B (codomain) is the image of atleast one element of A, then we say that f is a function of A 'onto' B.
Thus, f: A → B, such that f(A) = i.e., Range = Codomain Note Every polynomial function f: R → R of degree odd is onto.
Number of Onto (surjective) **Functions** Let A and B are finite sets having m and n elements respectively, such that $1 \leq n \leq m$, then number of onto (surjective) functions from A to B is ${}^n\Sigma_r = 1 (- 1)^{n-r}\, {}^nC_r r^m$ = Coefficient of^n in $n! (e^x - 1)^r$ If f : A → B is such that there exists atleast one element in codomain which is not the image of
Thus, f : A → B, such that $f(A) \subset B$
i.e., Range ⊂ Codomain

Important Points to be Remembered

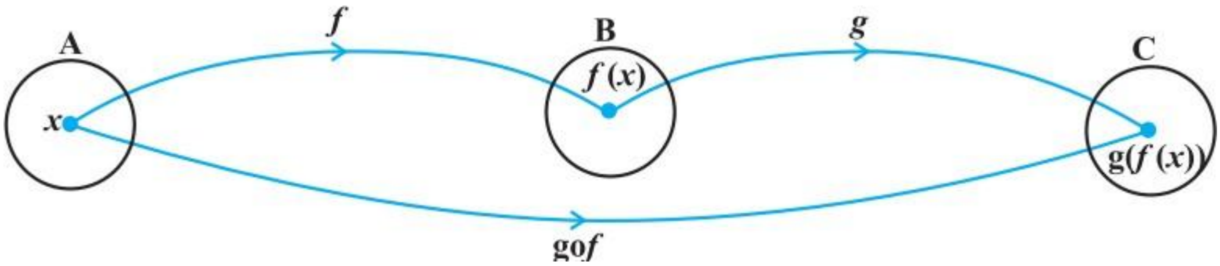
(i) If f and g are injective, then fog and gof are injective.
(ii) If f and g are surjective, then fog is surjective.
(iii) Iff and g are bijective, then fog is bijective.

Composition of Functions and Invertible

Function Composite Function

- Let f: A → B and g: B → C be two functions.
- Then the composition of f and g, denoted by $g \circ f$, is defined as the function $g \circ f$: A → C given by

  $g \circ f(x) = g(f(x)), \forall x \in A.$



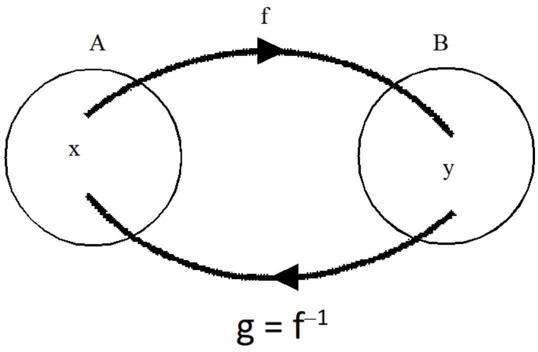
- Eg
  :
  - Let f : {2, 3, 4, 5} → {3, 4, 5, 9} and g : {3, 4, 5, 9} → {7, 11, 15} be functions
  - Defined as f (2) = 3, f (3) = 4, f (4) = f (5) = 5 and g (3) = g (4) = 7 and g (5) = g (9) = 11.
  - Find go f.
  - Solution
    - g o f(2) = g (f(2)) = g (3) = 7,
    - g o f (3) = g (f(3)) = g (4) = 7,
    - g o f (4) = g (f(4)) = g (5) = 11 and
    - g o f (5) = g (5) = 11

- It can be verified in general that gof is one-one implies that f is one-one. Similarly, gof is onto implies that g is onto.

- While composing f and g, to get gof, first f and then g was applied, while in the reverse process of the composite gof, first the reverse process of g is applied and then the reverse process of f.
- If f: X → Y is a function such that there exists a function g: Y → X such that gof = IX and fog = IY, then f must be one-one and onto.

Invertible Function

- A function f: X → Y is defined to be invertible, if there exists a function g: Y → X such that gof = IX and fog = IY. The function g is called the inverse of f
- Denoted by f^{-1}.



- Thus, if f is invertible, then f must be one-one and onto and conversely, if f is one-one and onto, then f must be invertible.

Theorem 1

- If f : X → Y, g : Y → Z and h : Z → S are functions, then
    - h o (g o f) = (h o g) o f.

- Proof
  We
  have
    - h o (g o f) (x) = h(g o f (x)) = h(g(f(x))), ∀ x in X
    - (h o g) o f (x) = h o g (f(x)) = h (g (f(x))), ∀ x in X.

  Hence, h∘ (g∘f) = (h∘g) ∘ f

Theorem 2

- Let f: X → Y and g: Y → Z be two invertible functions.
- Then gof is also invertible with $(go\ f)^{-1} = f^{-1} o\ g^{-1}$
- Proof
    - To show that gof is invertible with $(g\ o\ f)^{-1} = f^{-1} o\ g^{-1}$, it is enough to show that $(f^{-1} o\ g^{-1})\ o\ (g\ o\ f)$ = IX and $(g\ o\ f)\ o\ (f^{-1} o\ g^{-1})$ = IZ.

Now, $(f^{-1} \circ g^{-1}) \circ (g \circ f) = ((f^{-1} \circ g^{-1}) \circ g) \circ f$, by Theorem 1

$= (f^{-1} \circ (g^{-1} \circ g)) \circ f$, by Theorem 1

$= (f^{-1} \circ I_Y) \circ f$, by definition of g^{-1}

$= I_X$

Similarly, it can be shown that $(g \circ f) \circ (f^{-1} \circ g^{-1}) = I_Z$

Binary Operations

Definitions:

- A binary operation ∗ on a set A is a function ∗ : A × A → A. We denote ∗ (a, b) by a ∗ b.
- A binary operation ∗ on the set X is called commutative, if a ∗ b = b ∗ a, for every a, b ∈ X
- A binary operation ∗ : A × A → A is said to be associative if (a ∗ b) ∗ c = a ∗ (b ∗ c), ∀ a, b, c, ∈ A.
- A binary operation ∗ : A × A → A, an element e ∈ A, if it exists, is called identity for the operation ∗, if a ∗ e = a = e ∗ a, ∀ a ∈ A.
    - Zero is identity for the addition operation on R but it is not identity for the addition operation on N, as 0 ∉ N.
    - Addition operation on N does not have any identity.
    - For the addition operation + : R × R → R, given any a ∈ R, there exists – a in R such that a + (– a) = 0 (identity for '+') = (– a) + a.
    - For the multiplication operation on R, given any a ≠ 0 in R, we can choose $\frac{1}{a}$ such that $a \times \frac{1}{a} = 1$ (identity for '×') $= \frac{1}{a} \times a$
- A binary operation ∗ : A × A → A with the identity element e in A, an element a ∈ A is said to be invertible with respect to the operation ∗, if there exists an element b in A such that a ∗ b = e = b ∗ a and b is called the inverse of a and is denoted by a^{-1}

MCQ

1. Let n(A) =m and n(B)=n. Then the total number of non empty relations that can be defined from A to B is:

(a) m^n (b) n^m-1 (c) mn-1 (d) $2^{mn}-1$

2. Let A={ 1, 2, 3}. The total number of distinct relation which can be defined over A is:

(a) 6 (b) 8 (c) 2^9 (d) None of these

3. Let A={1, 2, 3, 4} and R={ (2,2), (3,3) ,(4,4), (1,2)} be relation on A. Then A is:

(a)reflexive (b) symmetric (c) transitive (d) None of these

4. The void relation on a Set A is:

(a) reflexive (b) symmetric and transitive

(c) reflexive and symmetric (d) reflexive and transitive

5. The relation 'is subset of' on the power set P(A) of a set A is:

(a) symmetric (b) anti-symmetric

(c) equivalence relation (d) None of these

6. The relation 'congruence modulo m' is :

(a) reflexive only (b) symmetric only

(c) transitive only (d) an equivalence relation.

7. Let R be the relation over the set N×N and is defined by (a,b) R (c,d)$\Longrightarrow$ a+d=b+c. Then R is:

(a) reflexive only (b) symmetric only

(c) transitive only (d) an equivalence relation.

8. Let P={(x,y): $x^2+y^2=1$, x,yϵ R}. Then P is:

(a) reflexive (b) symmetric (c) transitive (d) anti-symmetric

9. Let R be a relation on a Set A such that $R=R^{-1}$. Then R is

(a) reflexive (b) symmetric (c) transitive (d) None of these.

10. Let a relation R in the set of natural numbers be defined by (x,y)ϵ R$\Longleftrightarrow$ $x^2-4xy+3y^2=0$ for all x,yϵ N. Then the

relation R is :

(a) reflexive (b) symmetric (c) transitive (d) an equivalence relation.

11. Let A= {1, 2, 3}. Then the relation R={(2,3) in A is:

(a) symmetric only (b) transitive only (c) Symmetric and transitive only (d) None of these

12. Two points A and B in a Place are related if OA=OB. Where O is a fixed point. This relation is:

(a) reflexive but not symmetric (b) reflexive but not transitive

(c) equivalence relation (d) partial order relation

13. The relation R defined in A={1, 2, 3} by a R b if $|a^2-b^2|\leq 5$. Which of the following is not true?

(a) Domain of R={1, 2, 3}

(b) Range of R={5}

(c) $R=R^{-1}$

(d) R={(1,1), (2,2), (3,3), (2,1), (1,2), (2,3), (3,2)}

14. Let R be a relation in the set of natural numbers defined by $R=\{(1+x,1+x^2) : x\leq 5, x\in N\}$. Which of the following

is false

(a)Domain of R={2, 3, 4, 5, 6}
(b) Range of R={ 2, 5, 10 , 17, 26}
(c)R={ (2,2), (3,5) ,(4,10), (5,17) ,(6,26)}
(d) At least one is false.

15. The domain of the function f is defined by $f(x)=\frac{x^2+2x+1}{x^2-x-6}$ is given by:

(a) R-{3,-2} (b) R-{-3,2} (c) R-[3, -2] (d) R-(3,-2)

16. The domain and range of the function f given by :f(x)=2-|x-5| is:

(a) Domain=R^+ , Range=$(-\infty,1]$

(b) Domain=R, Range=$(-\infty,2]$

(c) Domain =R , Range=$(-\infty,2)$

(d) Domain= R^+, Range=$(-\infty,2]$

17. The domain for which the function defined by: $f(x)=3x^2-1$ and $g(x)=3+x$ is equal to:

(a)$\left\{-1,\frac{4}{3}\right\}$ (b) $\left\{-1,-\frac{4}{3}\right\}$ (c)$\left\{1,\frac{4}{3}\right\}$ (d) $\left\{1,-\frac{4}{3}\right\}$

18.if f(x)= $1-\frac{1}{x}$ then $f[f\left(\frac{1}{x}\right)]$ is:

(a) $\frac{1}{x-1}$ (b) $\frac{x}{x-1}$ (c)$\frac{1}{1+x}$ (d) $\frac{1}{x}$

19.The composite mapping fog of the maps f: R→R, $f(x)=\sin x, g: R\rightarrow R, g(x)=x^2$ is:

(a) $sin\ x^2$ (b) $(\sin x)^2$ (c)$\sin x+x^2$ (d) $\frac{\sin x}{x^2}$

20. Which of the following function is a polynomial function?4

(a) $\frac{2x^2+7x+4}{3}$ (b) $2x^2+x^{\frac{2}{3}}+4$ (c)$\frac{x^2-1}{x+4}, x\neq-4$ (d) $x^4+x^3+3x^2-7x+\sqrt{2}x^{-2}$

21. Which of the following is a rational function?

(a) $\frac{1}{3}\sqrt{4x^3+4x+7}$ (b) $\frac{3x^2-7x+1}{x-2}, x\neq 2$

(c)$\frac{3x^5+5x^3+2x+7}{x^{\frac{3}{2}}}, x>0$ (d) $\frac{\sqrt{1+x}}{2+5x}, x\neq-\frac{2}{5}$

22. Which of the following function is an even function

(a) $f(x)=\frac{a^x+a^{-x}}{a^x-a^{-x}}$ (b) $f(x)=\frac{a^x+1}{a^x-1}$ (c) $f(x)=x\frac{a^x-1}{a^x+1}$ (d)$f(x)=log_2(x+\sqrt{x^2+1})$

23. which of the following functions is an onto function?

(a) $f(x)=\sqrt{1+x+x^2}+\sqrt{1-x+x^2}$ (b) $f(x)=x\frac{a^x+1}{a^x-1}$

(c) $f(x)=\log\left(\frac{1-x}{1+x}\right)$ (d) f(x)=k(constant).

24. The Period of $\frac{|\sin x|+|\cos x|}{|\sin x-\cos x|}$ is:

(a) 2π (b) π (c)$\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

25. Which of the following function from A={x:-1≤x≤1} to itself is a bijection?

(a) $f(x)=|x|$ (b) $f(x)=x^2$ (c)$f(x)=\frac{x}{2}$ (d) $f(x)=sin\left(\frac{\pi x}{2}\right)$

26. if f is any function , then $\frac{1}{2}[f(x) + f(-x)] is\ always$

(a) one-one (b) neither even nor odd (c) even (d) odd

27. Which of the following function is not onto?

(a) $f: R \to R, f(x) = 3x + 5$

(b) $f: R \to R^+, f(x) = x^2 + 4$

(c) $f: R^+ \to R^+, f(x) = \sqrt{x}$

(d) None of these

ANSWERS

1	**d**	2	**c**	3	**c**	4	**b**	5	**b**	6	**d**	7	**d**	8	**b**	9	**b**
10	**a**	11	**b**	12	**c**	13	**b**	14	**c**	15	**a**	16	**b**	17	**a**	18	**b**
19	**a**	20	**a**	21	**b**	22	**c**	23	**c**	24	**b**	25	**d**	26	**c**	27	**b**